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Stochastic Modeling of Human Fertilization Using Uniform Distribution

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ABSTRACT: This study has develop a stochastic model for human fertilization for getting the insights of success and failure rates of human fertilization. The interaction of male and female gametes is modeled to understand the increase/decrease of success rate of fertilization. The model building was carried out by assuming the life times of sperm cell and ovule (egg) are random variable and follows Rectangular distribution. An index variable Z is defined by considering the events of success and failure as 1 and 0 respectively. Mathematical relations for different statistical measures are derived by using the joint transition probability distribution based on the random variable Z. This model has a good scope for better understanding on various issues of human fertilization using mathematical biology as a back end theory.

Keywords: Human fertilization, Stochastic Modeling, Uniform Distribution.

I. INTRODUCTION

Fertilization process takes place through the interaction of male gamete (sperm) and female gamete (egg). Sperm cells are movable and eggs are non-movable. During coitus process penis releases semen which contain around 300 million sperms which will enter into vagina. These sperms will share equal amount of X chromosome and Y chromosome. Though the sperm cell entered into the vagina it won't be fertilized. The sperm need to move across the Cervix which is the mouth of the uterus. It will be opened merely in the ovulation days of women or else it will be closed [3]. The sperms which entered into the cervix will quickly move over the uterus and it reaches the ampullary-Isthmic junction of the fallopian tube. During this passage of sperm from vagina to fallopian tube many million sperm cells will die. Only few thousands of sperms will remain active [2].

In the same time the ovulation process takes place which means the release of an egg from one of the ovaries it will often occur in the middle of the menstrual cycle. Egg or oocyte will contain only X chromosome. Egg travel through the fallopian tube because of the contraction happen in the tube. As soon as egg gets into the ampullary-Isthmic Junction, many sperm cells will try to fuse with egg. While interaction the sperm will undergo capacitation and acro some reaction [6]. The first sperm which contact the egg will get fertilized. The egg will release chemical that push other sperms to go outside the contact of egg. Over 300 million sperm, many millions will die in the journey only few will reach the egg, in that only one sperm will fuse with egg to fertilize and form zygote [5].

A Markov chain model was developed for calculations of the ovulation day of women and its effect on the baby sex [8]. A mathematical model was developed for studying the association of amenorrhea (absence of periods) dependence on gestation period (time between the conception and birth of a baby) [4]. Mathematical modeling was carried out and described the how the transient fecundable and fecundable states on the success/failure of fertilization [1].

Infertility is the real life problem in world. Infertility happens because of men, women, or both of them are important. Proper and successful interaction of sperm and egg reduce the infertility and increase the success rate of fertilization [7].

Fertilization happens when the interaction of male gamete sperm to the female gamete oocyte (egg) to form a zygote. In this process, penis release millions of sperm cells in the vagina and it move towards the egg cell but only few will reach the destination and the remaining will die in the journey, The life span of sperm is only 5 days and the life span of egg is only 24 hours so joining of sperm and oocyte are considered to be random and probabilistic.

This study surveyed the joint probability distribution of random variables specified as the life time of sperm cell $(X, \tau_{11} + \alpha < X \leq \tau_{12})$ and life time of egg $(Y, \tau_{21} + \beta < Y \leq \tau_{22})$. Considering they are independent and follows rectangular (Uniform) distributions. The time duration of active reproduction periods for sperm and egg are assumed as $'\alpha'(0 < \alpha \leq X)$ and $'\beta'(0 < \beta \leq Y)$ respectively. This section should be succinct, with no subheadings.

II. STOCHASTIC MODEL

This section deals with the development of a new stochastic model of sperm and egg fertilization of systematic arrangement of various biological issues through mathematical assumptions. The joint process of human fertilization will explain with appropriate probabilistic models.

A. Assumptions and Postulates of the model

This stochastic model will be explained with various mathematical biology assumptions on the mechanism of sperm cells and egg cells in the process of human fertilization. The success is labeled with successful mating of egg and sperm cell in Fallopian tube. Or else the event is considered to be failure of fertilization in the non-mating of sperm and egg. Here, the random variable Z will be equal to 1 if the successful mating of male and female gamete of human fertilization is happened and Z is considered to be 0 when there is no successful mating. Hence the overall event of the model is revealed with indexing variable 0 and 1. The successful fertility will be indexed with number 1 and non-successful fertilization will be indexed with 0. The joint probability with these indexed variable values is constructed. Statistical measures derived with the developed probability distribution will explain the behaviour of the phenomenon more elaborately.

Let $'X'$ be a random variable which is the life span of the sperm (male gamete). Let τ_{11} be the initial period of release of sperm in vagina. Let τ_{12} be the final period of release of the sperm for reproduction. Let $'\alpha'$ be the life span of the sperm cell during which it is able to participate in reproduction. Let $'Y'$ be a random variable that represent the life span of egg (female gamete). Let τ_{21} be the initial period (beginning time) of the ovulated egg. Let τ_{22} be the end period of the ovulated egg. Let $'\beta'$ be the ovulation time of women when egg is released either it go for fertilization or it will be disappeared. $\tau_1 = \tau_{12} - \tau_{11}$ and $\tau_2 = \tau_{22} - \tau_{21}$.

III. PROBABILITY MODEL

By considering uniform (Rectangular) distribution

$$P(Z=0) = P[\{(\tau_{11}+\alpha) \le X \le \tau_{12}\} \cap \{(\tau_{21}+\beta) \le Y \le \tau_{22}\}]$$

$$= \left(\int_{\tau_{11}+\alpha}^{\tau_{12}} \frac{dx}{(\tau_{12}-\tau_{11})}\right) \cap \left(\int_{\tau_{21}+\beta}^{\tau_{22}} \frac{dx}{(\tau_{22}-\tau_{21})}\right)$$

$$= 1 - \left[\frac{\beta(\tau_{12}-\tau_{11}) + \alpha(\tau_{22}-\tau_{21}) - \alpha\beta}{(\tau_{12}-\tau_{11})(\tau_{22}-\tau_{21})}\right]$$

$$P(z=1) = \left[\frac{\alpha(\tau_{22}-\tau_{21}) + \beta(\tau_{12}-\tau_{11}) - \alpha\beta}{(\tau_{12}-\tau_{11})(\tau_{22}-\tau_{21})}\right]$$

A. Mathematical Formula for Statistical Measures

Case1: when $\alpha \ne \beta$ and $\tau_1 \ne \tau_2$

In 1st case, we considered that the life span of sperm α is not equal to the life span of egg β and its reproduction time of sperm τ_1 and egg τ_2 are also not equal then their average and variance success rate of human fertilization is given below.

$$\text{Mean} = \frac{\beta\tau_1 + \alpha\tau_2 - \alpha\beta}{\tau_1\tau_2}$$

$$\text{Variance} = \frac{\beta\tau_1 + \alpha\tau_2 - \alpha\beta}{(\tau_1\tau_2)^2}[(\tau_1-\alpha)(\tau_2-\beta)]$$

The third and fourth central moments of the above distribution is given below

$$\mu_3 = \frac{\beta\tau_1 + \alpha\tau_2 - \alpha\beta}{(\tau_1\tau_2)^3}[(\tau_1\tau_2)^2 - 3(\tau_1\tau_2)(\beta\tau_1 + \alpha\tau_2 - \alpha\beta) + 2(\beta\tau_1 + \alpha\tau_2 - \alpha\beta)^2]$$

$$\mu_4 = \frac{\beta\tau_1 + \alpha\tau_2 - \alpha\beta}{(\tau_1\tau_2)^4}[(\tau_1\tau_2)^3 - 4(\tau_1\tau_2)^2(\beta\tau_1 + \alpha\tau_2 - \alpha\beta) + 6(\tau_1\tau_2)(\beta\tau_1 + \alpha\tau_2 - \alpha\beta)^2 - 3(\beta\tau_1 + \alpha\tau_2 - \alpha\beta)^3]$$

The skewness and kurtosis measures are given below

$$\beta_1 = \frac{(2\alpha\beta - 2\beta\tau_1 - 2\alpha\tau_2 + \tau_1\tau_2)^2}{(\beta\tau_1 + \alpha\tau_2 - \alpha\beta)(\tau_1-\alpha)(\tau_2-\beta)}$$

$$\beta_2 = \frac{(\tau_1\tau_2)^3 - 4(\tau_1\tau_2)^2(\beta\tau_1 + \alpha\tau_2 - \alpha\beta) + 6(\tau_1\tau_2)(\beta\tau_1 + \alpha\tau_2 - \alpha\beta)^2 - 3(\beta\tau_1 + \alpha\tau_2 - \alpha\beta)^3}{\left((\tau_1-\alpha)(\tau_2-\beta)\right)^2(\beta\tau_1 + \alpha\tau_2 - \alpha\beta)}$$

Case2: when $\alpha = \beta = \alpha$ and $\tau_1 \ne \tau_2$

In 2nd case, we considered that the reproductive time of sperm and egg is not equal but the life span of both the sperm and egg are equal then average and variance success rate of fertilization is given below.

$$\text{Mean} = \frac{\alpha(\tau_1 + \tau_2 - \alpha)}{\tau_1\tau_2}$$

$$\text{Variance} = \frac{\alpha(\tau_1 + \tau_2 - \alpha)}{(\tau_1\tau_2)^2}\left[\left(\tau_1\tau_2 - \alpha(\tau_1 + \tau_2 - \alpha)\right)\right]$$

The third and fourth central moment of the distribution is given below

$$\mu_3 = \frac{\alpha(\tau_1 + \tau_2 - \alpha)}{(\tau_1\tau_2)^3}[(\tau_1\tau_2)^2 - 3\alpha(\tau_1\tau_2)(\tau_1 + \tau_2 - \alpha) + 2\alpha^2(\tau_1 + \tau_2 - \alpha)^2]$$

$$\mu_4 = \frac{\alpha(\tau_1 + \tau_2 - \alpha)}{(\tau_1\tau_2)^4}[(\tau_1\tau_2)^3 - 4\alpha(\tau_1\tau_2)^2(\tau_1 + \tau_2 - \alpha) + 6\alpha^2(\tau_1\tau_2)(\tau_1 + \tau_2 - \alpha)^2 - 3\alpha^3(\tau_1 + \tau_2 - \alpha)^3]$$

The Shaping and peakedness measure are given below

$$\beta_1 = \frac{\left((\tau_1\tau_2)^2 - 3\alpha(\tau_1\tau_2)(\tau_1 + \tau_2 - \alpha) + 2\alpha^2(\tau_1 + \tau_2 - \alpha)^2\right)^2}{\alpha(\tau_1 + \tau_2 - \alpha)\left(\tau_1\tau_2 - \alpha(\tau_1 + \tau_2 - \alpha)\right)^3}$$

$$\beta_2 = \frac{(\tau_1\tau_2)^3 - 4\alpha(\tau_1\tau_2)^2(\tau_1 + \tau_2 - \alpha) + 6\alpha^2(\tau_1\tau_2)(\tau_1 + \tau_2 - \alpha)^2 - 3\alpha^3(\tau_1 + \tau_2 - \alpha)^3}{\alpha(\tau_1 + \tau_2 - \alpha)\left(\tau_1\tau_2 - \alpha(\tau_1 + \tau_2 - \alpha)\right)^2}$$

Case 3: When $\alpha \ne \beta$ and $\tau_1 = \tau_2 = \tau$

In 3rd case the life span of sperm and egg is not equal but the reproductive time of sperm and egg is equal then their average and variance are given below.

$$\text{Mean} = \frac{\tau(\alpha + \beta) - \alpha\beta}{\tau^2}$$

$$\text{Variance} = \frac{\tau(\alpha + \beta) - \alpha\beta}{\tau^4}[\tau^2 - \tau(\alpha + \beta) + \alpha\beta]$$

The moments of the above given model is demonstrated below

$$\mu_3 = \frac{\tau(\alpha + \beta) - \alpha\beta}{\tau^6}[\tau^4 - 3\tau^3(\alpha + \beta) + 3\tau^2\alpha\beta + 2(\tau(\alpha + \beta) - \alpha\beta)^2]$$

$$\mu_4 = \frac{\tau(\alpha + \beta) - \alpha\beta}{\tau^8}[\tau^6 - 4\tau^5(\alpha + \beta) + 4\tau^4\alpha\beta + 6\tau^2(\tau^2(\alpha + \beta)^2 + \alpha^2\beta^2 - \tau(\alpha + \beta)\alpha\beta)]$$

The Skewness and Kurtosis measures are

$$\beta_1 = \frac{(\tau^2 - 2\tau\alpha - 2\tau\beta + 2\alpha\beta)^2}{(\tau - \alpha)(\tau - \beta)(\tau\alpha + \tau\beta - \alpha\beta)}$$

$$\beta_2 = \frac{\tau^4 - 3\tau^3\alpha - 3\tau^3\beta + 3\tau^2\alpha^2 + 9\tau^2\alpha\beta + 3\tau^2\beta^2 - 6\tau\alpha^2\beta - 6\tau\alpha\beta^2 + 3\alpha^2\beta^2}{(\tau - \alpha)(\tau - \beta)(\tau\alpha + \tau\beta - \alpha\beta)}$$

Case4: When $\tau_1 = \tau_2 = \tau$ and $\alpha = \beta = \alpha$

In 4th case we considered both the reproductive time and life span of sperm and egg are equal and their statistical measures such as mean and variance are given below.

$$\text{Mean} = \frac{\alpha(2\tau-\alpha)}{\tau^2}$$

$$\text{Variance} = \frac{\alpha(\tau-\alpha)^2(2\tau-\alpha)}{\tau^4}$$

The moments of the above probability distribution are

$$\mu_3 = \frac{\alpha(\tau-\alpha)^2(2\tau-\alpha)(\tau^2-4\alpha\tau+2\alpha^2)}{\tau^6}$$

$$\mu_4 = \frac{\alpha(2\tau-\alpha)}{\tau^8}[(\tau-\alpha)^2(\tau^4-6\alpha\tau^3+15\alpha^2\tau^2-12\alpha^3\tau + 3\alpha^4)]$$

The coefficient of skewness and kurtosis measures are

$$\beta_1 = \frac{(\tau^2-4\alpha\tau+2\alpha^2)^2}{\alpha(\tau-\alpha)^2(2\tau-\alpha)}$$

$$\beta_2 = \frac{\tau^4-6\alpha\tau^3+15\alpha^2\tau^2-12\alpha^3\tau+3\alpha^4}{\alpha(2\tau-\alpha)}$$

IV. RESULTUS AND DISCUSSIONS

A. Graphs

In this section, the above given model is explained with simulated numerical data set and their graphical representation are given below.

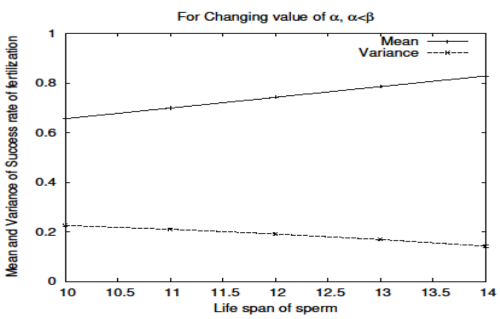

Fig. 1. Changing value of α, $\alpha < \beta$ and $\tau_1 \neq \tau_2$.

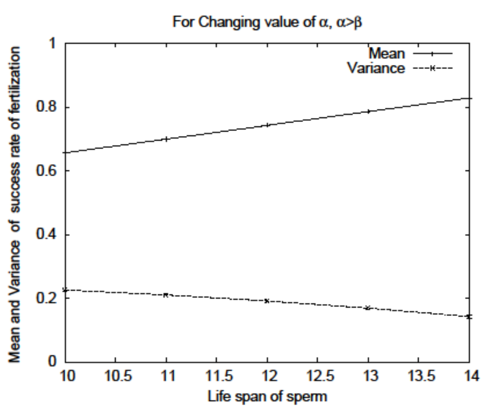

Fig. 2. Changing value of α, $\alpha > \beta$ and $\tau_1 \neq \tau_2$.

Figs. 1 and 2 represent the average and variance success rate of fertilization with respect to the life span of sperm and it is observed with two cases (i) For changing value of life span of sperm α which is less than the fixed value of life span of egg β. (ii) For changing value of life span of sperm αwhich is greater than the fixed value of life span of egg β. In both the cases the mean and variance are contradictory to each other. Positive correlation is observed between mean and sperm life span and negative correlation is notified between variance and sperm life span. Success rate of mean is increasing and variance is decreasing function which exposes that the healthy sperm life increases than their success rate of human fertilization also increases.

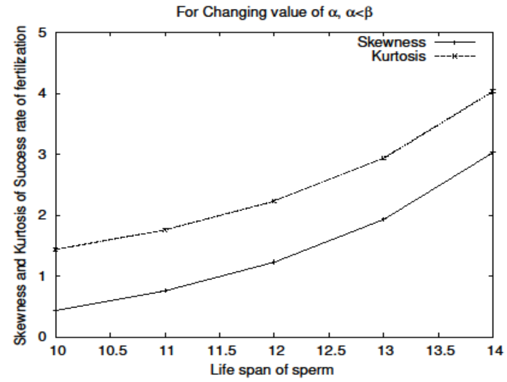

Fig. 3. Skewness & Kurtosis for $\alpha < \beta$.

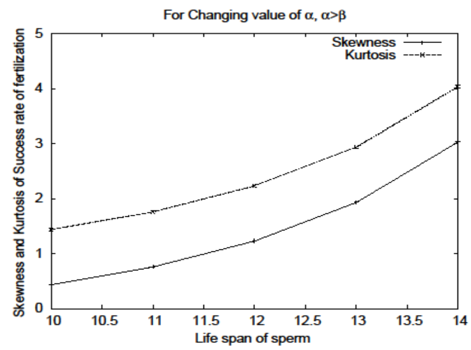

Fig. 4. Skewness & Kurtosis for $\alpha > \beta$.

Figs. 3 and 4 reveals the skewness and kurtosis measure for successful fertilization with respect to the sperm life span for two cases.(i) $\alpha < \beta$ which means for different values for the life span of sperm is less than the fixed value for the life span of egg. (ii) $\alpha > \beta$ which means for different values for the life span of sperm is greater than the fixed value for the life span of egg. In both the cases left skewness is observed which means model success rate is more than the mean success rate of human fertilization. Kurtosis measure increases when the sperm life span increases. It is noticed that when the durability and the life span of sperm increases then the chance of successful human fertilization also increases.

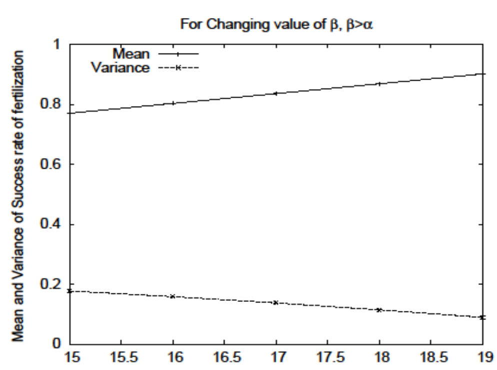

Fig. 5. Changing value of β, $\beta > \alpha$ and $\tau_1 \neq \tau_2$.

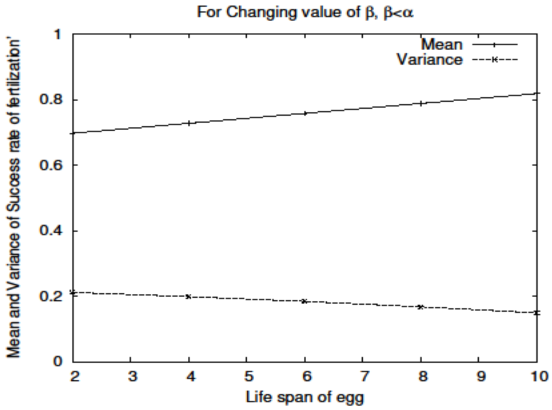

Fig. 6. Changing value of β, $\beta < \alpha$ and $\tau_1 \neq \tau_2$.

Figs. 5 and 6 exhibits the mean and variance of successful human fertilization associated with life time of egg and it is noticed with two cases (i) for different values of life span of egg β which is greater than the stable value of life span of sperm α. (ii) for different values of life span of egg β which is less than the stable value of life span of sperm α. In couple of cases the mean and variance are contrast to each other. Life span of egg is positively correlated with mean and negatively correlated with variance. Increasing function is noticed in mean success rate of fertilization and decreasing function is noticed in variance success rate of fertilization which tells when the life duration of egg increases than we find increment in human fertilization success rate.

Figs. 7 and 8 discloses the shaping and peakedness measures for the success rate of human fertilization linked with life span of egg for two cases.(i) $\beta > \alpha$ which means for varying values for egg life duration which is greater than the life span of sperm which is stationary. (ii) $\beta < \alpha$ Which means for varying values for egg life duration which is less than the sperm life span which is stationary. The shape of skewness in both the cases is left skewed. It is identified that the mean success rate is less than the model success rate of fertilization. Durability and life time of egg increases than the Kurtosis measure also increases then their chance for successful fertility is increased.

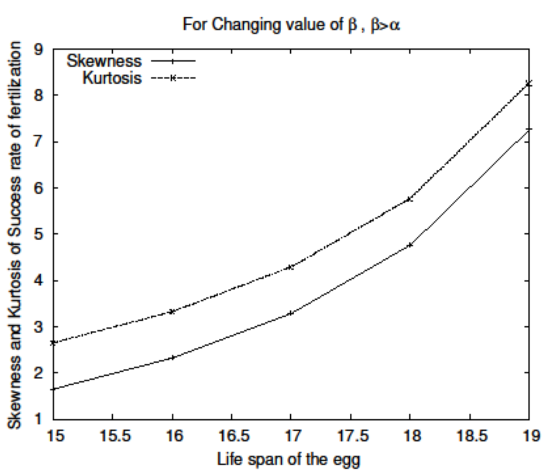

Fig. 7. Skewness & Kurtosis for $\beta > \alpha$.

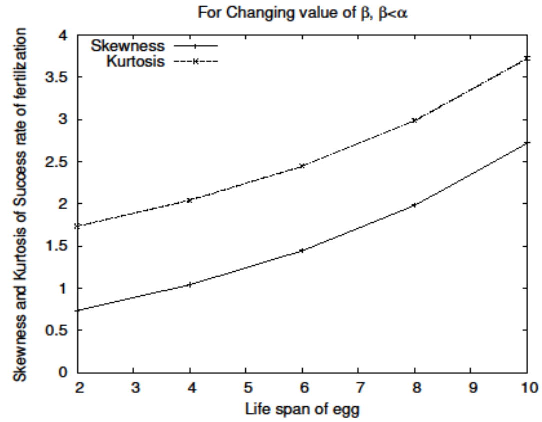

Fig. 8. Skewness & Kurtosis for $\beta < \alpha$.

V. CONCLUSION

This study develops a stochastic model for getting different indicators of human fertilization by formulating the biological processes with stochastic models. The mechanisms of increase or decrease of fertilization can be measured with the constructed models. Exploration of joint probability distribution through the index variable of success or failure of fertilization is the core area of the study. Assuming the simultaneous processes of male and female gametes happened on uniform scales, the combined action of successful mating and failure of mating is obtained. The joint probability distribution is obtained with the help of indexed variable Z taking the values zero and one which denotes the failure and successful mating of male and female gamete. The active stay times of sperm and egg cells in the reproductive process are assumed to with parameters alpha and beta. The model behavior is analyzed with different derived statistical measures in the cases of varying values of parameters α and β. The sensitivity analysis was carried out with some numerical data sets and presented the reports in figures for better understanding of model. The Pearson's coefficients based on moments are obtained for getting various statistical measures namely mean, variance, shaping measures, kurtosis measures, etc.

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